

Gauge-Higgs Unification Approach

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Abstract. When the extra dimensional space is not simply-connected, dynamics of the AB phase in the extra dimension can induce dynamical gauge symmetry breaking by the Hosotani mechanism. This opens up a new way of achieving unification of gauge forces. It leads to the gauge-Higgs unification. The Hosotani mechanism can be established nonperturbatively by lattice simulations, in which measurements of the Polyakov line give a clue. (OU-HET 751/2012, 8 June 2012)

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I. Introduction

Unification of gauge forces is achieved by starting with larger symmetry at high energies than the directly observed symmetry at low energies. The symmetry is spontaneously broken, being reduced to the observed one. The symmetry breaking mechanism constitutes the backbone of the unification.

There are various ways to achieve spontaneous gauge-symmetry breaking. The most popular one is the Higgs mechanism. A scalar field develops a nonvanishing vacuum expectation value at the tree level which is not invariant under gauge transformations. In the Coleman-Weinberg mechanism radiative corrections to the effective potential of the respected scalar fields induce symmetry breaking in a theory with no dimensionful parameter at the tree level. In technicolor theory scalar condensates of fermion-antifermion pairs break the symmetry.

In higher dimensional gauge theory there appears another way of breaking gauge symmetry. Extra-dimensional components of gauge potentials, as a consequence of dynamics, can induce gauge symmetry breaking. It is called the Hosotani mechanism.

II. Hosotani mechanism

When the extra-dimensional space is not simply connected, there appears an Aharonov-Bohm (AB) phase along a non-contractible loop in the extra dimension. This AB phase is a part of physical degrees of freedom of gauge fields. Its value is dynamically determined. With a nontrivial value it leads to gauge symmetry breaking.[1, 2, 3]

(a) QCD at $T \neq 0$ v.s. on S^1

The idea of the Hosotani mechanism came by examining QCD at finite temperature ($T \neq 0$). Finite temperature field theory (for equilibrium) is equivalent to field theory with an imaginary time τ in an interval $(0, \beta = 1/k_B T)$ with boundary conditions that all bosonic (fermionic) fields are periodic (anti-periodic). The imaginary time has topology of S^1 . Gluons in $SU(N_c)$ gauge theory acquire effective masses at finite temperature $m^2 = \frac{1}{3}g^2 T^2 (N_c + \frac{1}{2}N_F)$ where N_F is the number of fermions in the fundamental representation. Quark-gluon plasma at $T \neq 0$ gives screening of gluon propagation.

Now consider QCD on $R^1(\text{time}) \times R^2 \times S^1(\text{space})$ where one spatial dimension is a circle S^1 with a circumference β . After Wick rotation of the time axis the theory is the same as QCD at $T \neq 0$ except that boundary conditions become less restrictive. One can impose a boundary condition $\psi(x, y + \beta) = e^{i\delta} \psi(x, y)$ for fermions. Nothing is wrong with imposing a periodic boundary condition $e^{i\delta} = 1$. It is an easy exercise to show that the effective gluon masses are

changed:

$$m^2 = \frac{1}{3}g^2T^2(N_c + \frac{1}{2}N_F) \rightarrow m^2 = \frac{1}{3}g^2T^2(N_c - N_F). \quad (1)$$

What happens if $N_F > N_c$? Does $m^2 < 0$ imply the instability of the vacuum? It turns out that $\langle A_y \rangle \neq 0$ in the true vacuum and $\langle A_y \rangle \neq 0$ can lead to gauge symmetry breaking in non-Abelian gauge theory.

(b) Dynamics of AB phases

Consider a gauge theory on a product of d -dimensional Minkowski spacetime M^d and a circle S^1 with a coordinate y and radius R . Finite temperature QCD in 4D corresponds to the $d = 3$ case. The relevant quantities for the vacuum structure are Aharonov-Bohm (AB) phases along S^1 :

$$W = P \exp \left\{ ig \int_C dy A_y \right\}. \quad (2)$$

In $SU(N)$ gauge theory eigenvalues of W are given by $\{e^{i\theta_1}, \dots, e^{i\theta_N}\}$, $\sum_{j=1}^N \theta_j = 0 \pmod{2\pi}$. Note that constant A_y is nontrivial. Eigenvalues of W are gauge invariant so that they cannot be gauged away. Even if θ_j 's give vanishing field strengths, they represent physical degrees of freedom, affecting physics at the quantum level. They are AB phases. If $e^{i\theta_j} \neq e^{i\theta_k}$, it leads to gauge symmetry breaking.

The values of θ_j 's are not at our disposal. They are dynamically determined, once the matter content in the theory is specified. The true vacuum corresponds to the global minimum of the effective potential $V_{\text{eff}}(\theta_j)$. At the tree level $V_{\text{eff}}(\theta_j)^{\text{tree}} = 0$, as field strengths vanish. At the quantum level it becomes nontrivial. Particles in M^d consist of Kaluza-Klein towers, the spectra of which typically take the form of $m_n(\theta_H) = R^{-1}(n + \theta_H/2\pi)$ (n : an integer). Here θ_H represents AB phases θ_j collectively. The spectrum depend on θ_H . The effective potential at 1-loop is

$$V_{\text{eff}}(\theta_H)^{1\text{loop}} = \sum \pm \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \sum_n \ln \left\{ p^2 + m_n(\theta_H)^2 \right\}. \quad (3)$$

It is remarkable that the θ_H -dependent part of $V_{\text{eff}}(\theta_H)$ is finite, being free of divergence for any d . [4, 5] As a consequence the global minimum of $V_{\text{eff}}(\theta_H)$ is unambiguously determined.

III. Gauge-Higgs unification

This opens up a new way of having dynamical gauge symmetry breaking. Dynamics of AB phases in extra dimensions can induce gauge symmetry breaking. Four-dimensional fluctuations of these phases correspond to 4D Higgs fields. The gauge-Higgs unification is achieved. Higgs fields are identified with a part of extra-dimensional components of gauge potentials.

(a) $SU(3)$ on $M^d \times S^1$

Consider $SU(N)$ gauge theory on $M^d \times S^1$. Let us suppose that all fields are periodic on S^1 . In terms of θ_j ($j = 1, \dots, N$) spectra of massless particles are given by

$$m_n = \begin{cases} \frac{1}{R} \left(n + \frac{\theta_j - \theta_k}{2\pi} \right) & \text{for adjoint rep.,} \\ \frac{1}{R} \left(n + \frac{\theta_j}{2\pi} \right) & \text{for fundamental rep.} \end{cases} \quad (4)$$

Once the matter content is specified, $V_{\text{eff}}(\theta_H)$ in (3) is evaluated. For $d = 4$ the effective potential is given by

$$V_{\text{eff}}(\theta) = C \left\{ -3 \sum_{j,k=1}^N h_5 \left(\frac{\theta_j - \theta_k}{2\pi} \right) + 4N_{\text{fund}}^F \sum_{j=1}^N h_5 \left(\frac{\theta_j - \beta_{\text{fund}}}{2\pi} \right) + 4N_{\text{ad}}^F \sum_{j,k=1}^N h_5 \left(\frac{\theta_j - \theta_k - \beta_{\text{ad}}}{2\pi} \right) \right\},$$

$$h_d(x) = \sum_{n=1}^{\infty} \frac{\cos 2\pi n x}{n^d}, \quad C = \frac{3}{4\pi^2} \frac{1}{(2\pi R)^4}. \quad (5)$$

Here N_{fund}^F and N_{ad}^F are the numbers of fermion multiplets in the fundamental and adjoint representations, respectively. β_{fund} and β_{ad} are the boundary condition parameters appearing in $\psi(x, y + 2\pi R) = e^{i\beta} \psi(x, y)$. In general, each multiplet of fermions can have distinct β .

It is tempting to apply this to GUT, as GUT symmetry is normally broken to the SM symmetry by a Higgs field in the adjoint representation. For instance, if $e^{i\theta_1} = e^{i\theta_2} = e^{i\theta_3} \neq e^{i\theta_4} = e^{i\theta_5}$ is realized in $SU(5)$ theory, then one obtains $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ gauge symmetry breaking. It turns out that $SU(5)$ symmetry remains unbroken if fermion multiplets come in only **5** and **10** with $\beta = 0$.

Nontrivial examples of symmetry breaking are found when there are fermions in the adjoint representation.[2, 4] The effective potential $V_{\text{eff}}(\theta_1, \theta_2)$ in $SU(3)$ gauge theory are displayed in Figure 1. In pure gauge theory $SU(3)$ symmetry is unbroken. When one adjoint fermion is added ($N_{\text{ad}}^F, N_{\text{fund}}^F = (1, 0)$), V_{eff} is minimized at $(\theta_1, \theta_2, \theta_3) = (0, \frac{2}{3}\pi, -\frac{2}{3}\pi)$ and its permutations. The symmetry is broken to $U(1) \times U(1)$. When $(N_{\text{ad}}^F, N_{\text{fund}}^F) = (1, 1)$, V_{eff} is minimized at $(\theta_1, \theta_2, \theta_3) = (0, \pi, \pi)$ and its permutations. The symmetry is broken to $SU(2) \times U(1)$.

We note that the boundary conditions are $SU(3)$ symmetric in these examples. Dynamics of the AB phases induce symmetry breaking.

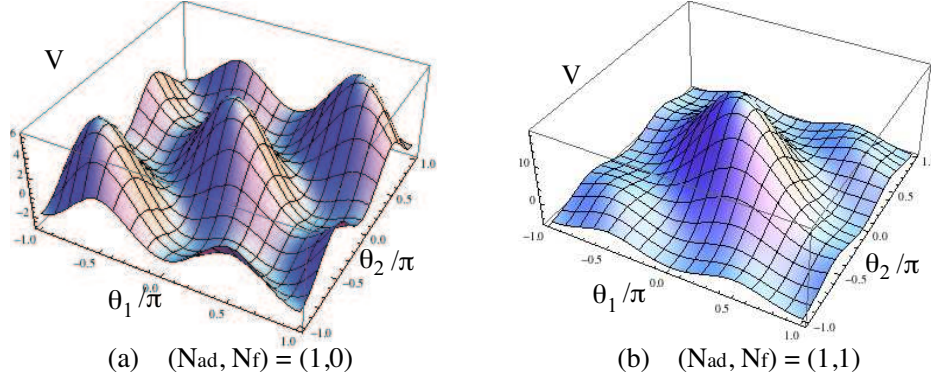


FIGURE 1. The effective potential $V_{\text{eff}}(\theta_1, \theta_2)$ in $SU(3)$ gauge theory on $M^4 \times S^1$ with massless fermions. (a) With $N_{\text{ad}}^F = 1, N_{\text{fund}}^F = 0$. (b) With $N_{\text{ad}}^F = N_{\text{fund}}^F = 1$. For both cases $\beta_{\text{ad}} = \beta_{\text{fund}} = 0$. The symmetry is broken to (a) $U(1) \times U(1)$, and (b) $SU(2) \times U(1)$.

(b) Electroweak unification and GUT

We have seen that non-Abelian gauge symmetry can be dynamically broken by the Hosotani mechanism. It is interesting to apply this mechanism to electroweak unification and GUT. To have realistic models one has to incorporate chiral fermions, which becomes highly nontrivial in higher dimensional gauge theory. One powerful way to have chiral fermions is to consider models in which extra-dimensional space is an orbifold. The simplest example of orbifolds is S^1/Z_2 . Two points y and $-y$ on S^1 are identified. There appear two fixed points at $y = 0$ and πR on S^1 , which are customarily called as two branes.

Many years ago Pomarol and Quiros formulated the standard model on $M^4 \times (S^1/Z_2)$. [6] Since then many models have been proposed. [7]-[19] With intensive experiments going on at LHC, which report possible candidates for the Higgs boson, it becomes necessary to make definitive predictions to be tested.

The most promising model of gauge-Higgs unification for electroweak interactions is the $SO(5) \times U(1)_X$ model in the Randall-Sundrum warped space. [9]-[12] $SO(5) \times U(1)_X$ breaks down to $SO(4) \times U(1)_X$ by the orbifold boundary conditions, to $SU(2)_L' \times U(1)_Y'$ by brane dynamics, and to $U(1)_{\text{EM}}$ by the Hosotani mechanism. It has been shown that the dynamical EW symmetry breaking takes place thanks to the presence of the top quark. The most striking result is that the Higgs boson, with a mass predicted around 130 GeV, becomes absolutely stable. [14, 16] The effective potential $V_{\text{eff}}(\theta_H)$ is minimized at $\theta_H = \frac{1}{2}\pi$. There emerges new parity (H -parity) under which the Higgs boson is odd, while all other SM particles are even.

Historically the Hosotani mechanism was first applied to GUT models.[1] On orbifolds the doublet-triplet splitting problem in $SU(5)$ GUT can be naturally solved.[20] Having chiral fermions and GUT symmetry breaking by the Hosotani mechanism simultaneously, however, is nontrivial.[21, 22] It is also known that boundary conditions at the fixed points of orbifolds fall into equivalence classes. [23, 24] Apparently different boundary conditions lead to the same physics as a consequence of dynamics of AB phases if those boundary conditions belong to the same equivalence class. New proposals for GUT have been made in Ref. [25].

IV. Nonperturbative Hosotani mechanism

So far the Hosotani mechanism for dynamical gauge symmetry breaking has been established only in perturbation theory. It is important to establish it nonperturbatively. We would like to describe how to do it on lattice.

There have already been lattice studies which, as explained below, support the Hosotani mechanism.[26]-[30] Myers and Ogilvie studied $SU(3)$ and $SU(4)$ gauge theories at finite temperature with periodic boundary conditions for fermions. Depending on fermion content, they claimed that there appear new phases.[27] Cossu and D’Elia investigated $SU(3)$ gauge theory on $16^3 \times 4$ lattice with massive fermions in the adjoint representation, examining Polyakov lines. They found phase transitions separating “confined”, “deconfined”, “split” and “re-confined” phases.[28] We show that all these results can be understood well with the notion of the Hosotani mechanism.

(a) Polyakov line and V_{eff}

Let us consider $SU(3)$ gauge theory on $M^3 \times S^1$ with massive fermions. The Wilson line W in (2) along S^1 has three eigenvalues, $(e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3})$ where $\sum \theta_j = 0$. The Polyakov line is

$$P = \frac{1}{3} \text{Tr} W = \frac{1}{3} \sum_{j=1}^3 e^{i\theta_j} = \frac{1}{3} \left\{ e^{i\theta_1} + e^{i\theta_2} + e^{-i(\theta_1+\theta_2)} \right\}. \quad (6)$$

In lattice simulations $\langle P \rangle$ is measured, which includes all quantum fluctuations. Corresponding to $M^3 \times S^1$, the simulations are done on $N_1^3 \times N_2$ lattice ($16^3 \times 4$ in ref. [28]).

The parameters in the continuum theory are g (gauge coupling), R (the radius of S^1), and m (fermion mass). Those in the lattice theory are β (lattice gauge coupling), a (lattice spacing) and am . The fermion mass m plays an important role in the lattice simulation. In the continuum theory V_{eff} at the one loop level takes the form $V_{\text{eff}}^{1\text{loop}} = g^2 R^{-3} f(\theta_1, \theta_2, \kappa)$ where $\kappa = \pi R m$.

$V_{\text{eff}}(\theta_1, \theta_2)$ is nontrivial. In the strong coupling regime (in the confinement phase), or for sufficiently large R , quantum fluctuations are large. All values of (θ_1, θ_2) are almost equally taken so that $\langle P \rangle = 0$. In the lattice simulations $\langle P \rangle$ should be centered around the origin in the complex plane.

In the weak coupling regime dominant gauge configurations are localized around one of the minima of $V_{\text{eff}}(\theta_1, \theta_2)$. Here the fermion mass as well as the gauge coupling becomes important. The method to evaluate V_{eff} with massive fermions has been developed in Ref. [31]. $V_{\text{eff}}^{1\text{loop}}$ with one adjoint fermion with a mass $\kappa = \pi R m = 0.55$ is depicted in Fig. 2. Notice that there develops more structure in V_{eff} than in the massless case in Fig. 1.

The wave function of the AB phases θ_1, θ_2 has finite spreading. It implies that the magnitude $|\langle P \rangle|$ get smaller than the value evaluated at the minimum of V_{eff} . In $SU(3)$ theory the phase of $\langle P \rangle$ are nontrivial and one can see a transition from one minimum to another. This is exactly what has been observed in the lattice simulations.

(b) Classification of phases

Examination of $V_{\text{eff}}^{1\text{loop}}$ with a given fermion mass shows that minima of V_{eff} are always located at some specific points. This behavior seems to persist to all order in perturbation theory, and seems to be supported by lattice simulations.

Phases classified by the Polyakov line are listed in Table 1. In each category the values of $(\theta_1, \theta_2, \theta_3)$ can take permutations of the given value. In the phase C, for instance, there are six degenerate global minima of V_{eff} . (A_1, A_2, A_3)

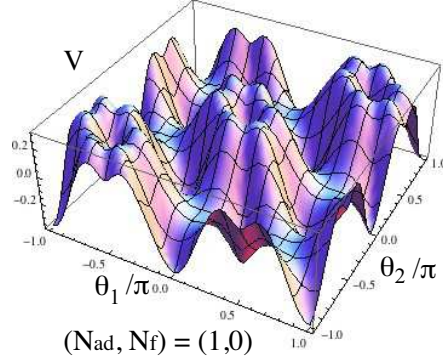


FIGURE 2. The effective potential $V_{\text{eff}}(\theta_1, \theta_2)$ in $SU(3)$ gauge theory with one adjoint fermion with $\kappa = \pi Rm = 0.55$.

and (B_1, B_2, B_3) form Z_3 multiplets. If there is no fermion in the fundamental representation, then the three phases in each Z_3 multiplet are degenerate.

TABLE 1. Classification of various phases in $SU(3)$ gauge theory. Location of the minima of V_{eff} , Polyakov line P , and residual symmetry are listed.

Phase	$(\theta_1, \theta_2, \theta_3)$	P	Symmetry	Names used in ref. [28]
X	large fluctuations	0	$SU(3)$	confined
$A_1, A_{2,3}$	$(0, 0, 0), (\pm \frac{2}{3}\pi, \pm \frac{2}{3}\pi, \pm \frac{2}{3}\pi)$	$1, e^{\pm 2i\pi/3}$	$SU(3)$	deconfined
$B_1, B_{2,3}$	$(\pi, \pi, 0), (\pm \frac{1}{3}\pi, \pm \frac{1}{3}\pi, \mp \frac{2}{3}\pi)$	$-\frac{1}{3}, \frac{1}{3}e^{\pm i\pi/3}$	$SU(2) \times U(1)$	split
C	$(0, \frac{2}{3}\pi, -\frac{2}{3}\pi)$	0	$U(1) \times U(1)$	re-confined

In lattice simulations the values of the coupling β and the fermion mass times lattice spacing ma are varied. V_{eff} at the 1 loop in the continuum theory is evaluated with varying $\kappa = \pi Rm$. We have plotted the values of V_{eff} in various phases in Fig. 3. One can infer the pattern of the phase transitions:

$$\begin{aligned}
 \text{Case (a)} \quad (N_{\text{ad}}, N_{\text{fund}}) = (1, 0): \quad & X \Leftrightarrow A \Leftrightarrow B \Leftrightarrow C \\
 \text{Case (b)} \quad (N_{\text{ad}}, N_{\text{fund}}) = (1, 1): \quad & X \Leftrightarrow A_{2,3} \Leftrightarrow B_1.
 \end{aligned} \tag{7}$$

Cossu and D'Elia have observed the same transition pattern in their lattice simulation.[28] Apparently large κ corresponds to large R , which in turn corresponds to small $|V_{\text{eff}}|$. Fluctuations due to gauge interactions become more important, and therefore it effectively corresponds to large gauge coupling. More investigation is necessary to pin down the phase structure.

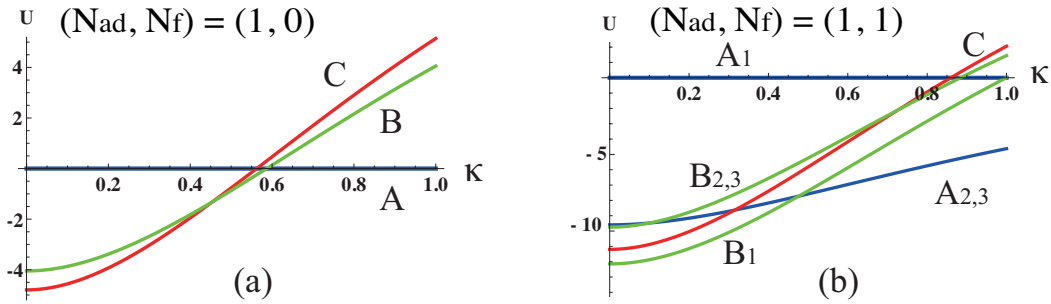


FIGURE 3. The value of the effective potential $V_{\text{eff}}^{1\text{loop}}$ in various phases in $SU(3)$ gauge theory as a function of $\kappa = \pi Rm$. (a) $(N_{\text{ad}}, N_{\text{fund}}) = (1, 0)$. (b) $(N_{\text{ad}}, N_{\text{fund}}) = (1, 1)$.

V. Summary

The above result indicates that dynamical gauge symmetry breaking by the Hosotani mechanism is taking place in $SU(3)$ gauge theory on $M^3 \times S^1$ when there are fermions in the adjoint representation. The lattice studies are mostly done in four dimensions to avoid the convergence issue in higher dimensions. If the Hosotani mechanism works in higher dimensions nonperturbatively, it gives a new paradigm for unifying gauge forces.

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